

Mass Renormalization and Chiral Symmetry Breaking in Nonlinear Spinor Theory

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The chiral invariance of conventional nonlinear spinor theory is assumed to be broken. This results in a finite mass renormalization of the nucleon mass. The self-consistency of the assumption is demonstrated. The meson mass eigenvalue equation is solved in lowest approximation. The spectrum which is no longer parity degenerate is in reasonable agreement with experiment.

1. Introduction

Nonlinear spinor theory is an attempt for a unified description of high energy phenomena¹. It claims to calculate self-consistently all particle masses and coupling constants. So no (bare) mass term for the fundamental field $\psi(x)$ is introduced into the field equation: all masses are assumed to arise purely dynamically. Therefore, the propagator of the ψ -field is supposed to have a pole at a finite mass value corresponding to the nucleon.

But there is one problem associated with this line of reasoning: Due to the absence of a mass term, the field equation has as an additional symmetry property the chiral transformations, i.e. it is invariant under the substitution

$$\psi(x) \rightarrow \exp \{i \alpha \gamma_5\} \psi(x).$$

So the nucleon is forced to stay massless if the chiral invariance is retained. The situation is similar to the vanishing photon mass in quantum electrodynamics which is related to the gauge invariance of the theory.

A possibility to render the nucleon massive in nonlinear spinor theory is to associate a particle with opposite parity to the nucleon. Then we have a couple of particles degenerate in mass, and chiral invariance is restored. This recipe was used in nonlinear spinor theory in the past². The price one has to pay is not only the introduction of an experimentally unobserved parity partner of the nucleon, but also the mesonic spectrum turns out to be parity degenerate. So the bound states usually identified with the π - or η -meson are, in reality, mixtures of scalar and pseudoscalar states. This remedy seems to us not too attractive either.

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A way out of this dilemma is offered by the assumption of a spontaneous breaking of chiral invariance by the vacuum. We will follow this approach in the present paper. In fact, this idea is not new. It was used in a similar model in³, and we take a great deal of our philosophy from this work. But there are some distinctive features of nonlinear spinor theory, e.g. the finiteness of all integrals arising from the indefinite metric. Furthermore, the conclusions are very different from those of³ due to a slightly different form of the interaction.

Already in the past, nonlinear spinor theory has been compared with models taken from solid state physics. We mention only⁴ where the analogy to a semi-conductor model has been stressed since in both cases one is dealing with the dynamics of "dressed" particles or clusters. The nucleons were related to the polarons and the mesons to the excitons i.e. to polaron-antipolaron bound states.

Following³, we will now propose an analogy of nonlinear spinor theory to superconductivity. The nucleon is considered as a mixture of bare fermions with opposite chirality, but the same charge, just like the quasi-particle in a superconductor which is a mixture of bare electrons with opposite charge but the same spin. In both cases we find a spontaneous breakdown of a symmetry group which manifests itself in a finite nucleon mass κ or a finite gap parameter φ . If these two quantities vanish, the respective particles are eigenstates of chirality and electric charge. The mesons are compared to collective excitations of quasi-particle pairs. We will not pursue this analogy further mainly because of lack of competence.

As is well known, the spontaneous breakdown of a symmetry is combined with the occurrence of a Goldstone particle (for a review see e.g.⁵). In³ a massless (pseudoscalar) meson was found which



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could be identified with the Goldstone particle. We do not find such a particle. This failure might be due to the indefinite metric invalidating the Goldstone theorem. But the reason might be also the unsymmetrical treatment of the nucleon and the meson eigenvalue equation (second order in the coupling constant for the nucleon, first order for the meson). Future work must be devoted to this question.

Before we start, we have to say something on the functional formulation of nonlinear spinor theory⁶. It is needed for performing dynamical calculations, especially for the scattering problem. Since we are mainly dealing with symmetry properties we may avoid the use of the complicated formalism of functional quantum theory. For our purposes we need only the dynamical equations which were derived in⁷. The resulting equations are of conventional Bethe-Salpeter form and do not require the understanding of the whole apparatus of functional quantum theory.

The paper is organized as follows: In Section 2 we introduce the symmetry breaking and discuss the relation to mass-renormalization. In Section 3 we solve the nucleon eigenvalue equation and demonstrate the consistency of the assumption of broken symmetry. The boson mass eigenvalue equation is solved in lowest approximation in Section 4. Finally, we give some concluding remarks.

2. Mass Renormalization and Symmetry Breaking

We will now start with the discussion of chiral symmetry breaking and its relation to mass renormalization. In order to keep the algebra as simple as possible, we shall use the non-Hermitean representation and disregard the isospin. These degrees of freedom will be introduced later on. Of course we will employ the four-component Dirac representation which is parity-symmetric. The field equation is then:

$$i \gamma^\mu \partial_\mu \psi(x) + \kappa_0 \psi(x) = \varepsilon \frac{l^2}{2} [\gamma^\mu \psi(x) \bar{\psi}(x) \gamma_\mu \psi(x) + \gamma_5 \gamma^\mu \psi(x) \bar{\psi}(x) \gamma_5 \gamma_\mu \psi(x)]. \quad (2.1)$$

We have added a (bare) mass term $\kappa_0 \psi(x)$ as we want to separate clearly the effects of mass renormalization and of symmetry breaking. Later on, we will return to the usual theory with $\kappa_0 = 0$ and exhibit the special problems related to this question. Our field equation differs from the model of³ in the relative sign of the vector and axial-vector inter-

action. The overall sign of the interaction $\varepsilon = \pm 1$ will be fixed in Section 4.

By the requirement of invariance against Lorentz transformations including reflections the propagator of the field $\psi(x)$ is restricted to the Lehmann spectral representation:

$$F(p) = \int_0^\infty dm^2 \left[\frac{\varrho_1(m^2)}{\gamma p - m + i\varepsilon} + \frac{\varrho_2(m^2)}{\gamma p + m + i\varepsilon} \right]. \quad (2.2)$$

Since $\psi(x)$ is somehow associated to the nucleon field, $F(p)$ will have a pole at the (physical) nucleon mass κ . So the spectral function will contain a contribution $\sim \delta(m^2 - \kappa^2)$ and it is essential that this pole occurs in only one of the two spectral functions, since there is no particle with the same mass as the nucleon but with opposite parity.

The bare mass κ_0 is related to the physical mass κ by

$$\kappa = \kappa_0 + \delta\kappa \quad (2.3)$$

where the mass renormalization $\delta\kappa$ is caused by the interaction. In fact, the requirement $\delta\kappa = 0$ will lead immediately to the free theory $l^2 = 0$. In nonlinear spinor theory, $\delta\kappa$ will be finite due to the indefinite metric of the state space: we do not impose the usual positivity conditions on $\varrho_i(m^2)$.

If we want to do a dynamical calculation in nonlinear spinor theory we have to resort to Tamm-Dancoff-like approximations, since the canonical Bethe-Salpeter methods do not work for indefinite metric⁸. So we will encounter not only the exact propagator F in the Feynman graphs, but also the Green's function G of the Dirac operator with the bare mass κ_0 . It is the hope that finally G will be dressed to F as it is done in⁹, but nobody knows how to do this in practice in nonlinear spinor theory.

This short-coming may be remedied, however, by performing the mass-renormalization in (2.1). So we add $\delta\kappa \psi(x)$ on both sides which leads to the equation:

$$i \gamma^\mu \partial_\mu \psi(x) + \kappa \psi(x) = \delta\kappa \psi(x) + \varepsilon \frac{l^2}{2} [\gamma^\mu \psi(x) \bar{\psi}(x) \gamma_\mu \psi(x) + \gamma_5 \gamma^\mu \psi(x) \bar{\psi}(x) \gamma_5 \gamma_\mu \psi(x)]. \quad (2.4)$$

If we now invert the left-hand side Dirac operator the Green's function G will have the pole at the physical mass κ as the exact propagator F . Of course, G will still differ from F (it does not contain continuum contributions nor regularizing pole terms). We will

First we discuss the Fermion eigenvalue equation. It allows the self-consistent determination of the physical nucleon mass. In lowest approximation the equation reads:

$$-\text{Diagram} = \text{Diagram} + 6 \cdot \text{Diagram} \quad (3.1)$$

The calculation may be looked upon as a generalized Hartree-Fock procedure: The self-energy is calculated perturbatively with propagators which are already subject to the self-energy effect. Explicitly, (3.1) reads:

$$\tau_\sigma = G_{\sigma\alpha'} (-i\lambda_2' \beta)_{\alpha'\beta'} \kappa \tau_{\beta'} + 6 G_{\sigma\alpha} V_{\alpha\beta\gamma\delta} G_{\beta\alpha'} F_{\gamma\gamma'} F_{\delta\delta'} V_{\alpha'\beta'\gamma'\delta'} \tau_{\beta'}. \quad (3.2)$$

This simplifies considerably when we perform the whole algebra and reduce with respect to the baryon number gauge group. The calculation is now more involved than in the chiral symmetric case, since both F and G contain two terms. For baryon number $B=1$ we find:

$$\tau(J) = (\gamma J + \kappa) \kappa g(J) \tau(J) - (\gamma J + \kappa) g(J) \left(\frac{\kappa l}{2\pi} \right)^4 \varepsilon^2 \frac{3}{2} \left\{ \frac{1}{3} (\gamma^e g^{\nu\lambda} + \gamma^\lambda g^{\nu e} + \gamma^\nu g^{e\lambda}) L_{\nu\lambda}(J) + \kappa K(J) \right\} \tau(J). \quad (3.3)$$

In (3.3) we have introduced the two nucleon self-energy integrals which arise from the two closed loops in (3.1):

$$L^{\nu\lambda}(J) = \frac{\kappa^4}{\pi^4} \int d^4p d^4q \frac{(J-p-q)^e p^\nu q^\lambda}{[(J-p-q)^2 - \kappa^2] p^4 (p^2 - \kappa^2) q^4 (q^2 - \kappa^2)}, \quad (3.4)$$

$$K(J) = \frac{\kappa^2}{\pi^4} \int d^4p d^4q \frac{1}{[(J-p-q)^2 - \kappa^2] p^2 (p^2 - \kappa^2) q^2 (q^2 - \kappa^2)}. \quad (3.5)$$

In (3.4) (3.5) we have omitted the imaginary parts $i\varepsilon$ in the denominators for brevity. We come back to the actual calculation of these integrals in a moment. In fact, $L^{\nu\lambda}(J)$ is found to be symmetric in the three Lorentz indices:

$$L^{\nu\lambda}(J) = (J^e g^{\nu\lambda} + J^\nu g^{e\lambda} + J^\lambda g^{e\nu}) L_1(\lambda) + \frac{1}{J^2} J^e J^\nu J^\lambda L_2(\lambda). \quad (3.6)$$

On dimensional reasons, $L_i(\lambda)$ will only depend on the dimensionless variable

$$\lambda = J^2/\kappa^2 \quad (3.7)$$

as well as $K(\lambda)$. With the definition

$$L(\lambda) = 2 L_1(\lambda) + L_2(\lambda) \quad (3.8)$$

(3.3) is finally transformed into:

$$\tau(J) = g(J) \left\{ (\gamma J + \kappa) \kappa \left[1 - \frac{3}{2} \left(\frac{\kappa l}{2\pi} \right)^4 \varepsilon^2 \left(\frac{\lambda+1}{2} L(\lambda) + K(\lambda) \right) \right] + (\gamma J - \kappa) \kappa \left[\frac{3}{2} \left(\frac{\kappa l}{2\pi} \right)^4 \varepsilon^2 \frac{\lambda-1}{2} L(\lambda) \right] \right\} \tau(J). \quad (3.9)$$

(3.9) is now multiplied by $J - \kappa$ and we go to the nucleon mass-shell $J^2 + \kappa^2$, $\lambda=1$. Since $\tau(J)$ is a solution of the Dirac equation

$$(\gamma J - \kappa) \tau(J) = 0 \quad \text{for } J^2 = \kappa^2 \quad (3.10)$$

we deduce from (3.9):

$$0 = \kappa \left\{ 1 - \frac{3}{2} \left(\frac{\kappa l}{2\pi} \right)^4 \varepsilon^2 [L(1) + K(1)] \right\} \tau(J). \quad (3.11)$$

Contrary to¹, we now have in addition a first order term arising from the mass-renormalization vertex. It should be noted that all tadpole graphs vanish due to the form of the interaction which is purely vector and axial vector. This is in contrast to³ where those terms had to be included.

For a non-trivial wavefunction $\tau(J)$, (3.11) has two solutions: Either $\kappa=0$ which is the usual perturbative solution, or we have a connection between the mass κ and the coupling constant l^2 :

$$1 = \frac{3}{2} \left(\frac{\kappa l}{2\pi} \right)^4 \varepsilon^2 [L(1) + K(1)]. \quad (3.12)$$

If (3.12) has an acceptable solution which must be established by the actual computation of $L(1)$ and $K(1)$ we have demonstrated the consistency of the

assumption of a non-invariant vacuum and of spontaneous symmetry breaking.

So we have to calculate the integrals (3.4) (3.5). In order to avoid unnecessary algebraic complica-

$$K(\lambda) = \int_0^1 du \, dx \, dy \, dz (1-u) [u z(1-z) + x z(1-u) + y(1-z)(1-u) - \lambda u(1-u)z(1-z)]^{-1}. \quad (3.13)$$

For calculating the mass-renormalization in (3.12) we need only the value of the function at $\lambda = 1$. It turns out that this special value is calculated much more simply than the whole function $K(\lambda)$. So we have confined us to this value. Then two of the parametric integrals are almost trivial and the others lead to some dilogarithms which offer no principal difficulties. The final result is

$$K(1) = \sqrt{3}\pi - (\pi^2/3) = 2.1515. \quad (3.14)$$

Similarly, we find

$$L(1) = \frac{1}{12} - \frac{\sqrt{3}}{24} \pi + \frac{\pi^2}{12} = 0.6791 \quad (3.15)$$

and we get for the physical mass:

$$\varepsilon^2 ((\kappa l)/(2\pi))^4 = 0.2355. \quad (3.16)$$

So we have indeed a physical solution in the broken symmetry theory. Compared to the chiral invariant case, where one finds

$$\varepsilon^2 ((\kappa l)/(2\pi))^4 = 1.068 \quad (3.17)$$

the effective coupling constant is much smaller in our case. The sign of the interaction is not fixed by this calculation, since we have only a second order contribution $\sim \varepsilon^2 = 1$. ε will be fixed in the next section.

Before we close this section we will make some remarks on the functional form of $L(\lambda)$ and $K(\lambda)$. Even if we have made no attempt to calculate it in general (which might be even impossible analytically⁹), we have calculated the low energy behaviour. For $\lambda \sim 0$ we find a linear divergence of $L(\lambda)$:

$$L(\lambda) \sim -\frac{1}{12\lambda} - \frac{5}{12} \ln \lambda + \dots \quad (3.18)$$

whereas $K(\lambda)$ stays finite:

$$K(0) = 7 - \frac{\pi^2}{6} - 2\sqrt{3}\text{Cl}_2\left(\frac{\pi}{3}\right) = 1.8392. \quad (3.19)$$

$\text{Cl}_2(\theta)$ denotes the second order Clausen function which is defined by¹²

$$\text{Cl}_2(\vartheta) = \sum_{n=1}^{\infty} (\sin n \vartheta / n^2). \quad (3.20)$$

tions we will exemplify the calculation on $K(\lambda)$. The denominators are combined with the usual Feynman trick and the momentum integrations are performed easily. This leads to the parametric integral:

The more singular behaviour of $L(\lambda)$ had to be expected because of the stronger regularization. Besides the low energy limit $\lambda \rightarrow 0$, the high energy behaviour (i.e. $\lambda \rightarrow \infty$) might be of interest for a self-consistent calculation of the propagator. But this turns out to be much more involved. It is not a simple power behaviour, but is modified by logarithmic terms in an unpleasant way. So we do not give the result here.

4. The Boson Eigenvalue Equation

The most important consequences of the spontaneous symmetry breaking show up in the spectrum of the bosons. Since the nucleon propagator is no longer degenerate with respect to parity, the same is true for the mesons and the deuteron. So we can really identify the calculated mass eigenvalues with the observed particles. Up to now this was not possible, strictly speaking, since all states were mixed up of a couple of opposite parity states.

The boson mass eigenvalue equation in nonlinear spinor theory has the general structure:

$$\text{Diagram} = \text{Diagram with } K_2 \text{ box} + \left(\text{Diagram with } K_1 \text{ box} + \text{Diagram with } K_1 \text{ box} \right) \left(1 + \text{Diagram with } K_2 \text{ box} + \dots \right) \text{Diagram} \quad (4.1)$$

where K_1 , K_2 denote one- and two-particle irreducible kernels, respectively. In a conventional Bethe-Salpeter equation, only the first term is present. The additional terms perform the dressing of G to the full propagator F . They are also present in the chiral invariant form of the theory. The only difference is that K_1 has now a first order contribution in addition to the usual second order graph:

$$-\boxed{K_1} = -\bullet \times + \frac{9}{4} \bullet \text{---} \bigcirc \text{---} + \dots \quad (4.2)$$

The main effect of these terms, at least at low energies, is the shift of the pole in G to the nucleon mass which we have taken into account from beginning. So we may safely disregard these self-energy terms altogether. For the two-particle irreducible kernel K ,

we use the lowest order approximation and get the eigenvalue equation

$$\text{Diagram} = \frac{3}{2} \left(\text{Diagram 1} \cdot \text{Diagram 2} \right) \quad (4.3)$$

We will not go through all the details of the calculation which has been described at many places in the past (e.g. in¹³). Since the interaction in (4.3) is separable, the problem reduces to an algebraic equation in momentum space after separation of the center of mass motion. We first discuss the results for mesons (baryon number $B=0$). They may have the isospin $T=0$ or $T=1$ and the parity $\eta_p = \pm 1$. The mass eigenvalues are found from the equation:

$$1 = \frac{1}{4} \left(\frac{\kappa l}{2\pi} \right)^2 \varepsilon[3 - T(T+1)] \{ 2 \eta_p D^{(0)}(\lambda) + 2 \lambda D^{(1)}(\lambda) - 2 D^{(2)}(\lambda) \} \quad (4.4)$$

for spin $j=0$, and from

$$1 = \frac{1}{4} \left(\frac{\kappa l}{2\pi} \right)^2 \varepsilon[3 - T(T+1)] \{ -2 \eta_p D^{(0)}(\lambda) - 2 \lambda D^{(1)}(\lambda) - 2 D^{(2)}(\lambda) \} \quad (4.5)$$

for spin $j=1$. The functions $D^{(i)}(\lambda)$, $i=1, 2, 3$, are derived from the loop integral by:

$$\begin{aligned} & \int \frac{d^4 q}{(2\pi)^4} \frac{(q-J)^\mu q^\nu}{((J-q)^2 - \kappa^2) q^4 (q^2 - \kappa^2)} \\ &= \frac{i}{(4\pi)^2} J^\mu J^\nu D^{(1)}(\lambda) + \frac{i}{2(4\pi)^2} g^{\mu\nu} \kappa^2 D^{(2)}(\lambda) \end{aligned} \quad (4.6)$$

$$\begin{aligned} & \int \frac{d^4 q}{(2\pi)^4} \frac{1}{((J-q)^2 - \kappa^2) q^2 (q^2 - \kappa^2)} \\ &= \frac{i}{(4\pi)^2} D^{(0)}(\lambda). \end{aligned} \quad (4.7)$$

The integrals are calculated easily by the introduction of Feynman parameters. It should be noted that the equations for $\eta_p = +1$ and $\eta_p = -1$ differ only in the sign of the $D^{(0)}(\lambda)$ -term which arises from the chiral symmetry breaking term. For $D^{(0)}(\lambda) = 0$ we will recover parity degeneracy of the solutions.

Additionally, the solutions are characterized by the charge conjugation eigenvalue $\eta_c = \pm 1$ (at least the neutral members of the isospin multiplets). Due to the contact nature of the interaction only the following combinations occur¹⁴:

$$\begin{aligned} j=0: & \quad \eta_p = +1, \eta_c = -1; \quad \eta_p = -1, \eta_c = +1; \\ j=1: & \quad \eta_p = +1, \eta_c = +1; \quad \eta_p = -1, \eta_c = -1. \end{aligned} \quad (4.8)$$

The local kind of interaction is also the reason for the absence of solutions with spin higher than $j=1$. For the combination of quantum numbers $j^{\eta_p \eta_c}$ the eigenvalue equation may be written in the form

$$1 + \frac{1}{4} ((\kappa l)/2\pi)^2 \varepsilon[3 - T(T+1)] q_j^{\eta_p \eta_c}(\lambda) = 0. \quad (4.9)$$

The functions $q_j^{\eta_p \eta_c}(\lambda)$ are derived from the calculation of the integrals (4.6) (4.7) and are given explicitly by:

$$q_0^{+-}(\lambda) = -\frac{2}{\lambda} - \frac{2}{\lambda^2} \ln|1-\lambda|, \quad (4.10)$$

$$\begin{aligned} q_0^{-+}(\lambda) = & -\frac{2}{\lambda} + \left(4 - \frac{4}{\lambda} - \frac{2}{\lambda^2} \right) \ln|1-\lambda| \\ & + (2\lambda - 8)P(1, 1, \lambda), \end{aligned} \quad (4.11)$$

$$\begin{aligned} q_1^{++}(\lambda) = & \frac{2}{3} + \frac{2}{3\lambda} + \left(-\frac{2}{3}\lambda + 4 - \frac{2}{\lambda} + \frac{2}{3\lambda^2} \right) \\ & \cdot \ln|1-\lambda| + \left(-\frac{\lambda^2}{3} + \frac{8}{3}\lambda - \frac{16}{3} \right) \cdot P(1, 1, \lambda), \end{aligned} \quad (4.12)$$

$$\begin{aligned} q_1^{--}(\lambda) = & \frac{2}{3} + \frac{2}{3\lambda} + \left(-\frac{2}{3}\lambda + \frac{2}{\lambda} + \frac{2}{3\lambda^2} \right) \\ & \cdot \ln|1-\lambda| + \left(-\frac{\lambda^2}{3} + \frac{2}{3}\lambda + \frac{8}{3} \right) \cdot P(1, 1, \lambda). \end{aligned} \quad (4.13)$$

We have introduced the abbreviation

$$P(\alpha, \beta, \gamma) = \frac{1}{\sqrt{A(\alpha, \beta, \gamma)}} \ln \frac{\gamma - \alpha - \beta - \sqrt{A(\alpha, \beta, \gamma)}}{\gamma - \alpha - \beta + \sqrt{A(\alpha, \beta, \gamma)}}, \quad (4.14)$$

$$A(\alpha, \beta, \gamma) = \alpha^2 + \beta^2 + \gamma^2 - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma.$$

The various functions $q_j^{\eta_p \eta_c}(\lambda)$ are shown in Fig. 1 for natural parity ($\eta_p = (-1)^j$) and in Fig. 2 for unnatural parity ($\eta_p = -(-1)^j$). For $\lambda=1$ the functions have a (logarithmic) singularity generated from the pinch of the massless double pole in F and the pole in G . The corresponding imaginary part for $\lambda>1$ has to be suppressed in order to avoid the possible decay of the bound states into this unphysical channel. This is achieved by taking the absolute values in the logarithms in (4.10) – (4.13). The cusp at $\lambda=4$ is generated by the pinch of the two massive poles in F and G , and the imaginary part of $P(1, 1, \lambda)$ for $\lambda>4$ describes the possible decay of the meson into two nucleons. The presence of this physical cut is an advantage of our approach.

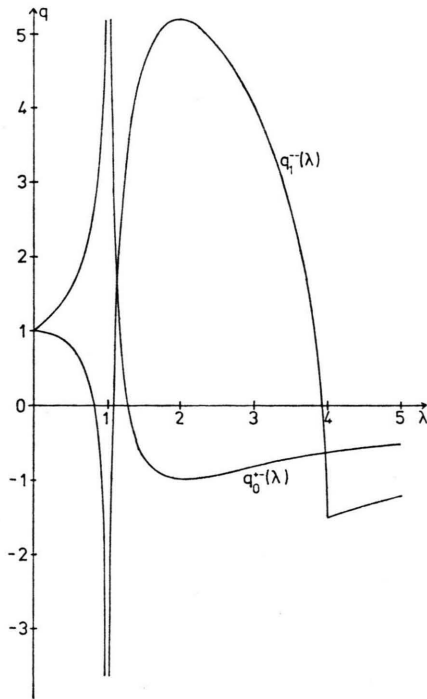


Fig. 1. The functions $q_0^{+-}(\lambda)$ and $q_1^{--}(\lambda)$, defined by (4.8), (4.11).

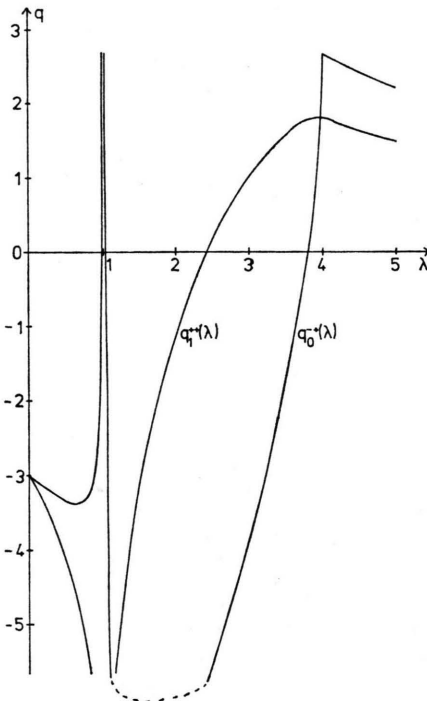


Fig. 2. The functions $q_0^{--}(\lambda)$ and $q_0^{++}(\lambda)$ defined by (4.9), (4.10). $q_0^{--}(\lambda)$ has a minimum at $q_0^{--}(1.43) = -8.46$.

Something must be said on the 0^{+-} -states. These are called exotic of the second kind in the quark model¹⁵, since this combination of quantum numbers is not possible nonrelativistically. No particles of this kind have been found experimentally so far. Hence we fix the sign of the interaction by the requirement that there are no bound states in the corresponding channel. This leads to the coupling constant

$$\varepsilon ((\kappa l)/(2\pi))^2 = +0.4853 \quad (4.15)$$

or $\varepsilon = +1$. This is in agreement with the sign used before¹. With the sign of the interaction fixed, we may now calculate immediately the mass eigenvalues for the other quantum number assignments from (4.11) – (4.13). The eigenvalues are listed in Table 1.

Before we may compare the results with experiment, we have to investigate the norm of the corresponding states i.e. whether the states are ghosts or physical particles. Since we now have a nucleon mass pole at each leg of the scattering matrix, we may safely employ the usual Bethe-Salpeter normalization¹⁶ which relates the norm of the states to the derivative of the corresponding q -function as follows: For physical particle it is:

$$j=0: dq/d\lambda > 0; \quad j=1: dq/d\lambda < 0. \quad (4.16)$$

The solutions belonging to ghosts according to (4.16) are given in brackets in Table 1. It should be noted that we find physical solutions in the vector case too, contrary to the chiral invariant theory. The experimental particles and their masses are taken

Table 1. The solution of the meson mass eigenvalue Equation (4.7) for the various quantum numbers T , j , η_p , and η_c for the coupling constant (4.13). Ghost solutions are given in brackets. The experimental masses are taken from¹⁷.

j	T	η_p	η_c	m [MeV]	Particle	exp. [MeV]
0	0	-1	+1	896	η	549
				(953)	η'	958
				1698	E	1420
				(1075)	π	140
0	1	-1	+1	1203	$D(?)$	1285
				—	—	—
				—	—	—
0	0	+1	-1	—	—	—
0	1	+1	-1	—	—	—
1	0	+1	+1	—	—	—
1	1	+1	+1	915	A_1	1100
				(962)	—	—
1	0	-1	-1	930	ω	784
				(948)	φ	1019
1	1	-1	-1	934	ϱ	770
				(944)	—	—

from¹⁷. The correspondence is not unique in all cases, since there are several resonances for the same quantum numbers in some cases.

The form of the spectrum is highly satisfactory. We find physical solutions for all quantum numbers which correspond to observed particles. We have no isoscalar 1^{++} -particle in agreement with experiment. The calculated masses have the correct order of magnitude with the possible exception of the 0^{-+} -channel if one wants to relate the solutions to the π - and η -meson. In this case the calculated masses are too large, especially for the π -meson. But there are other resonances in this channel for which the agreement is much better.

For the π - and η -meson, another explanation might be possible: With the spontaneous break-down of chiral invariance, a (massless) Goldstone boson must show up. It will be pseudoscalar in this case³ and so it might be possible to identify it with the π - and η -meson. We did not find such a state in our calculation. A reason for this failure can be the unsymmetrical treatment of fermion and boson eigenvalue equation: The boson mass operator is of first order in the coupling constant whereas the fermion mass operator is of second order, since the first order tadpole graph vanishes for our interaction. So we expect the Goldstone boson to show up in the second order boson eigenvalue equation. But it will be a difficult task to establish it because one has to resort to numerical computations².

Now we turn to the deuteron eigenvalue equation (baryon number $B=2$). By the interaction, the quantum numbers are restricted to $T=1$, $j=0$. So we get an equation only for the scalar deuteron which is realized in nature as an antbound state on the second sheet 65 keV above threshold. Both parities $\eta_p = \pm 1$ are possible relativistically, whereas nonrelativistically only $\eta_p = +1$ is found. A charge conjugation parity is not defined for $B=2$. The eigenvalue equation reads

$$1 + \frac{1}{4} \left(\frac{\kappa l}{2\pi} \right)^2 \varepsilon(-2 q_2^{\eta_p}(\lambda)) = 0 \quad (4.17)$$

(the factor -2 is conventional) with the functions:

$$q_2^+(\lambda) = 1 - \left(\lambda + \frac{3}{\lambda} \right) \ln|1-\lambda| - \left(\frac{\lambda^2}{2} - \lambda - 4 \right) \cdot P(1, 1, \lambda), \quad (4.18)$$

$$q_2^-(\lambda) = 1 - \left(\lambda - 6 + \frac{3}{\lambda} \right) \ln|1-\lambda| - \left(\frac{\lambda^2}{2} - 4\lambda + 8 \right) \cdot P(1, 1, \lambda). \quad (4.19)$$

These functions are shown in Figure 3.

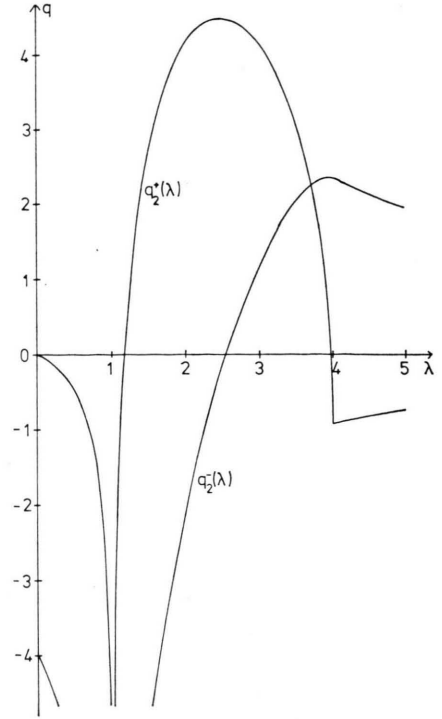


Fig. 3. The functions $q_2^+(\lambda)$ and $q_2^-(\lambda)$, defined by (4.16), (4.17).

For the sign of the interaction given by (4.15), we find no solution for the unphysical parity $\eta_p = -1$. We believe it to be a major success of our approach to avoid both the exotics of the second kind and the unphysical deuteron. For $\eta_p = +1$ we find a ghost at $\lambda = 1.98$ and a physical particle at $\lambda = 3.03$ which corresponds to a binding energy of 243 MeV. The binding effect turns out to be too large by orders of magnitude, but we could not expect a better agreement, since the binding potential of the deuteron is essentially long-ranged and can not be approximated by a δ -function.

5. Conclusions

The results of our work are quite encouraging. We have demonstrated the self-consistency of the assumption that chiral symmetry is broken spontaneously by the vacuum. It leads to a non-perturbative equation for the nucleon mass. The whole nucleonic and mesonic spectrum is no longer parity-degenerate. The calculated boson masses are of the correct order of magnitude. For some quantum numbers we did

not find bound states in agreement with experiment. The question of Goldstone bosons remains to be solved by a second order (numerical) calculation which is in preparation.

Our approach might also be useful in the scattering problem, since it has a pole at the physical nucleon mass at all external legs of the scattering matrix. Thus the use of more conventional methods seems possible at least in nucleon-nucleon scattering.

So one may hope that our work is a step towards the goal of confronting nonlinear spinor theory with experiment.

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